CODE - A

TEST ID 001917

NTPL

ALL INDIA TEST SERIES

JEE (Main) - 2019

FULL TEST - 2

Time : 3 Hours *Maximum Marks : 360*

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **90 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Maths).
10. The test is of **3 hours** duration. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

Name of the Candidate (in Capitals) ____________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

PHYSICS

SECTION – I

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. In the relation $P = \frac{\alpha}{\beta} e^{-\frac{\alpha Z}{k\theta}}$ P is pressure Z is distance k is Boltzman constant and θ is the temperature. The dimension formula of β will be.

(a) $M^0L^2T^0$ (b) $M^1L^2T^{-1}$ (c) ML^0T^{-1} (d) $M^0L^2T^1$

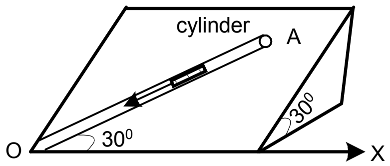
2. A body when projected vertically up covers a total distance D. During the time of its flight is t. If there were no gravity, the distance covered by it during the same time is equal to

(a) 0 (b) D (c) 2D (d) 4D

3. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity v and other with a uniform acceleration a. If α is the angle between the lines of motion of two particles then the least value of relative velocity will be at time given by

(a) $\frac{v}{a}\sin\alpha$ (b) $\frac{v}{a}\cos\alpha$ (c) $\frac{v}{a}\tan\alpha$ (d) $\frac{v}{a}\cot\alpha$

4. An inclined plane makes an angle 30° with the horizontal. A groove *OA* = 5m cut in the plane makes an angle 30° with *OX*. A short smooth cylinder is free to slide down the influence of gravity. The time taken by the cylinder to reach from *A* to *O* is (g = 10 m/s^2)



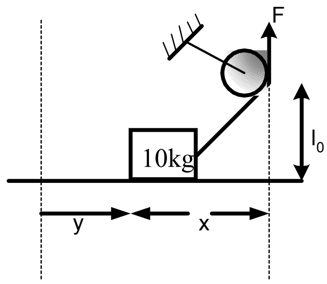
(a) 4 s

(b) 2 s

(c) $2\sqrt{2}$s

(d) 1 s

5. A block of mass 10kg is pulled by a force F having magnitude 20N. Find work done by force F if body moves 5m in right direction. Given that l_0 = 1 m and x = 25m initially.



(a) $= F\left[\sqrt{636} - \sqrt{401}\right]J$

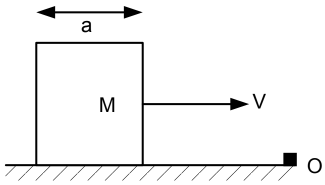
(b) $= F\left[\sqrt{626} - \sqrt{401}\right]J$

(c) $= F\left[\sqrt{646} - \sqrt{401}\right]J$

(d) $= F\left[\sqrt{696} - \sqrt{401}\right]J$

6. A tube-well pump out 2400kg of water per minute. If water is coming out with a velocity of 3m/s, the power of the pump is
   (a) 120W (b) 180 W (c) 240 W (d) 90 W

7. A cubical block of side a is moving with velocity V on a horizontal smooth plane as shown in Figure. It hits a ridge at point O. The angular speed of the block after it hits O is

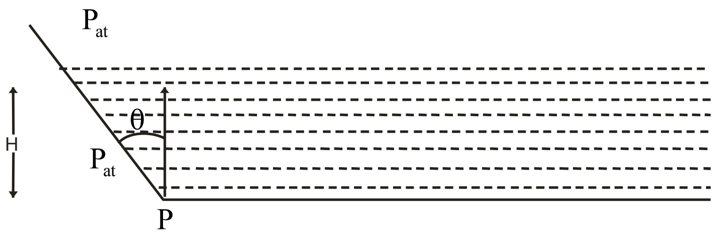


   (a) 3V/ (4a)
   (b) 3V/ (2a)
   (c) $\sqrt{3V}/\left(\sqrt{2a}\right)$
   (d) zero

8. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct?
   (a) (Angular momentum) is conserved about the centre
   (b) Only direction of angular momentum is conserved
   (c) It spirals towards the centre
   (d) Its acceleration is towards the centre

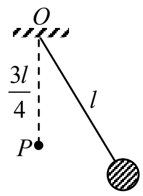
9. The radius of a planet is R. A satellite revolves around it in a circle of radius r with angular speed ω. The acceleration due to gravity on planet's surface will be:
   (a) $\frac{r^3\omega}{R}$ (b) $\frac{r^2\omega^3}{R}$ (c) $\frac{r^3\omega^2}{R^2}$ (d) $\frac{r^2\omega^2}{R}$

10. A wall of width ω at an angle θ with the normal is subject to water pressure in a vessel. The height of water in the vessel is H and ρ is the density of water. Then Force exerted on the wall is?



   (a) Average pressure on the wall is $2P_{at}+\frac{1}{2}\rho gH$
   (b) Average pressure on the wall is $\frac{1}{2}\left(P_{at}+\rho gH\right)$
   (c) Force exerted on wall is $\frac{1}{2}\left(P_{at}+\rho gH\right)\frac{\omega H}{\sin\alpha}$
   (d) Force exerted on the wall is $\left(P_{at}+\frac{1}{2}\rho gH\right)\frac{\omega H}{\cos\theta}$

11. A pendulum has time period T for small oscillations. An obstacle P is situated below the point of suspension O at a distance $\frac{3l}{4}$. The pendulum is released from rest. Throughout the motion the moving string makes small angle with vertical. Time after which the pendulum returns back to its initial position is.



   (a) T (b) $\frac{3T}{4}$
   (c) $\frac{3T}{5}$ (d) $\frac{4T}{5}$

12. A sound wave of wavelength λ travels towards the right horizontally with a velocity v. It strikes and reflects from a vertical plane surface, travelling at acceleration a starting from rest away from the source. The number of positive crests striking in a time interval of 5 sec on the wall is

(a) $5\left(v+\frac{5a}{2}\right)/\lambda$ (b) $5\left(v-\frac{5a}{2}\right)/\lambda$ (c) $5v/\lambda$ (d) $(v-5a)/5\lambda$

13. For an ideal gas:

(i) The change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas.

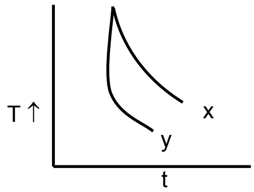
(ii) The change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.

(iii) The internal energy does not change in an isothermal process.

(iv) No heat is added or removed in an adiabatic process.

(a) i, ii (b) i, ii, iii (c) i, ii, iii, iv (d) i, iii

14. The graphs gives the variation of temperature of two bodies having the same surface area with time where A_x and A_y represent absorptivity and ε_x and ε_y represent emissivity then



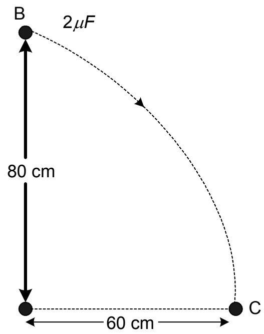
(a) $\varepsilon_x > \varepsilon_y$ and $A_x < A_y$

(b) $\varepsilon_x < \varepsilon_y$ and $A_x > A_y$

(c) $\varepsilon_x > \varepsilon_y$ and $A_x > A_y$

(d) $\varepsilon_x < \varepsilon_y$ and $A_x < A_y$

15. A charge of 2μC is brought from B to C along the path as shown by arrow in the figure. The work done is



(a) 0.75 J

(b) 0.6J

(c) 0.06J

(d) 0.075J

16. A ring has charge Q and radius R. If a charge q is placed at its centre then the increase in tension in the ring is

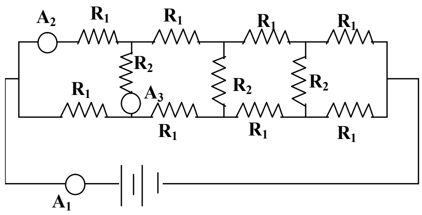
(a) $\frac{Qq}{4\pi\varepsilon_0 R^2}$ (b) zero (c) $\frac{Qq}{4\pi^2\varepsilon_0 R^2}$ (d) $\frac{Qq}{8\pi^2\varepsilon_0 R^2}$

17. In the Bohr model, the electron of a hydrogen atom moves in a circular orbit of radius 5.3×10^{-11} m with a speed of 2.2×10^6 m/s the current I in the orbit.

(a) 1.06 mA (b) 2.06 mA (c) 0.06 mA (d) 3.06 mA

18. In the given circuit, $R_1 = 10\Omega$, $R_2 = 6\ \Omega$ and E = 10 V.

Then reading of A_2



(a) Reading of A_2 is 1 amp

(b) Reading of A_2 is 2 amp

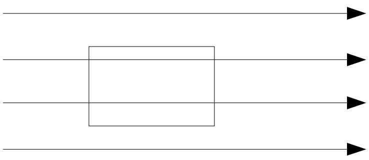
(c) Reading of A_2 is 1/4 amp.

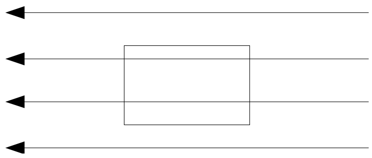
(d) Reading of A_2 is 3 amp

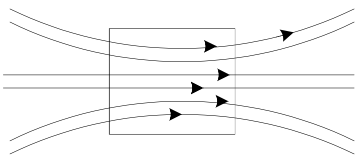
19. The time period of a small magnet in a horizontal plane is T. Another magnet B oscillates at the same place in a similar manner. The size of two magnets is the same but the magnetic moment of B is four times that of A. The time period of B will be

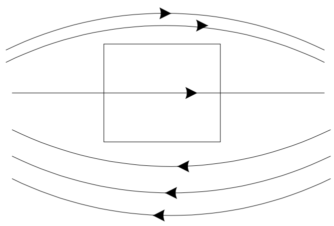
(a) $\frac{T}{4}$ (b) $\frac{T}{2}$ (c) 2T (d) 4 T

20. A uniform magnetic field is directed from left towards right in the plane of paper. When a piece of soft iron is placed parallel to the field. The magnetic lines of force passing through it will be

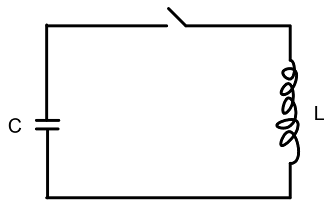
(a) 

(b) 

(c) 

(d) 

21. The Capacitor shown in the Fig. is initially charged to a voltage V and the circuit is then closed. The charge on C is found to oscillate with frequency ω. The frequency will be doubled if



(a) The voltage V is doubled

(b) Both L and C are halved

(c) Both L and C are doubled

(d) Both L and C are creased by a factor $\sqrt{2}$

22. In the middle of a long solenoid there is a coaxial ring of square cross-section, made of conductivity material of resistivity p. The thickness of the ring is equal to h, its inside and outside radii are equal to 'a' and 'b' respectively. What is the current induced in a radial width (dr), where the magnetic field varies with time as $B = \beta t$?

(a) $\frac{hdr\beta}{2p}$ (b) $\frac{hrdr\beta}{4p}$ (c) $\frac{hrdr\beta}{p}$ (d) $\frac{hrdr\beta}{2p}$

23. Power of a convex lens is + 5D (μ_g= 1.5). When this lens is immersed in a liquid of refractive index μ, it acts like a divergent lens of focal length 100 cm. Then refractive index of the liquid will be

(a) $\frac{5}{3}$ (b) $\frac{5}{4}$ (c) $\frac{6}{5}$ (d) none of these

24. Two slits S_1 and S_2 illuminated by a white light source give a white central maxima. A transparent sheet of refractive index 1.25 and thickness t_1 is placed in front of S_1. Another transparent sheet of refractive index 1.50 and thickness t_2 is placed in front of S_2. If central maxima is not effected, then ratio of the thickness of the two sheets will be

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

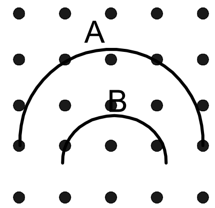
25. Two radioactive materials. X_1 and X_2 have decay constants 10 λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2. will be 1/e after a time

(a) $\frac{1}{10\lambda}$ (b) $\frac{1}{11\lambda}$ (c) $\frac{11}{10\lambda}$ (d) $\frac{1}{9\lambda}$

26. For a common base amplifier, the values of resistance gain and voltage gain are 3000 and 2800 respectively. The current gain will be:

(a) 0.93 (b) 0.83 (c) 0.73 (d) 0.63

27. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in the figure. Then



(a) $m_A v_A < m_B v_B$

(b) $m_A v_A > m_B v_B$

(c) $m_A < m_B$ and $v_A < v_B$

(d) $m_A = m_B$ and $v_A = v_B$

28. A photoelectric cell is illuminated by a small bright source of light placed at 1m. If the same source of light is placed 2m away, the electrons emitted by the cathode

(a) each carries one quarter of its previous momentum.

(b) each carries one quarter of its previous energy.

(c) are half the previous number.

(d) are one quarter of the previous number.

29. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately.

(a) 540 nm (b) 400 nm (c) 310 nm (d) 220nm

30. A circular wire of radius R carries a total charge Q distributed uniformly over its circumference. A small length of the wire subtending angle θ at the centre is cut off. Then the electric field at the centre due to the remaining portion will be.

(a) $=\frac{Q}{4\pi^2\varepsilon_0 R^2}\sin(\theta)$ (b) $=\frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{\theta}{2}\right)$ (c) $=\frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{\theta}{4}\right)$ (d) $=\frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{\theta}{8}\right)$

CHEMISTRY

SECTION – II

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

31. In HF, the inter nuclear distance is 92 pm. If the molecule has 43.5% ionic character, the observed dipole moment will be

(a) 4.42 D (b) 1.98 D (c) 0.98 D (d) 0.90 D

32. Which one of the following statements about benzyne intermediate is incorrect?

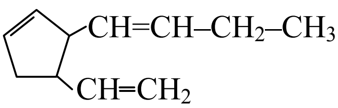
(a) All the carbon atoms are in same plane.

(b) It is Anti aromatic because it contains 10 π electrons.

(c) The π bond in benzyne is due to lateral overlap of sp^2 hybrid orbitals.

(d) All the above statements are correct.

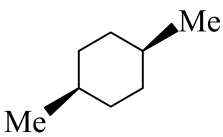
33. Total number of stereo isomers possible for the following compound,

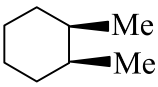


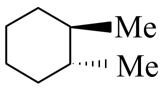
are

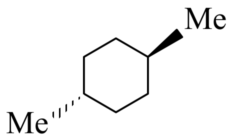
(a) 2 (b) 4 (c) 8 (d) 16

34. Which one of the following isomeric structures has the lowest energy?

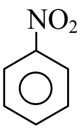
(a) 

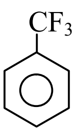
(b) 

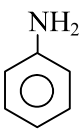
(c) 

(d) 

35. Friedal–Craft's acylation is preferred for which of the following molecule?

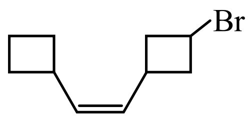
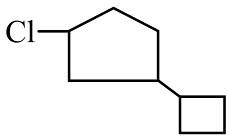
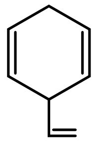
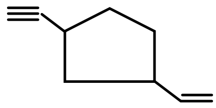
(a) 

(b) 

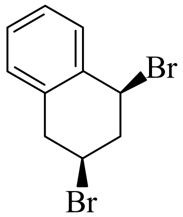
(c) 

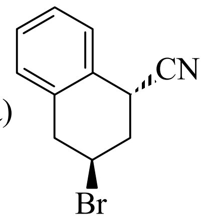
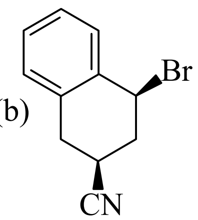
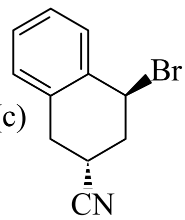
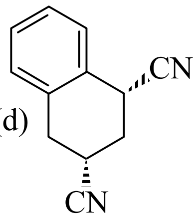
(d) none of these

36. The compounds having index of un-saturation = 4, are

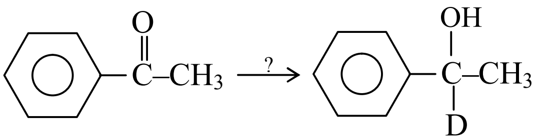
(i)  (ii)  (iii)  (iv) 

(a) (i), (iii) (b) (i), (iv) (c) (iii), (iv) (d) (i), (iii), (iv)

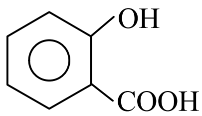
37.  $\xrightarrow{\text{1eq NaCN}}$ (A) Product (A) is:

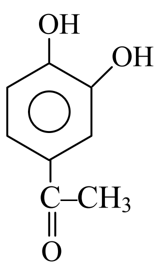
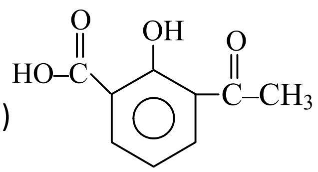
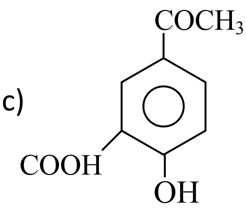
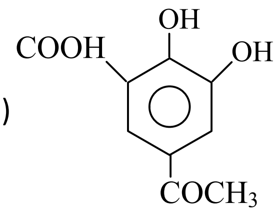
(a)  (b)  (c)  (d) 

38. Which of the following reagents would carry out the following?



(a) $NaBD_4$ in CH_3OH (b) $LiAlH_4$, then D_2O (c) $NaBD_4$ in CH_3OD (d) $LiAlD_4$, then D_2O

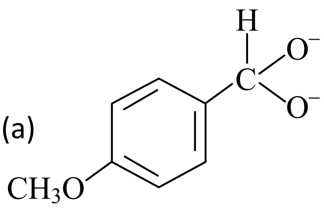
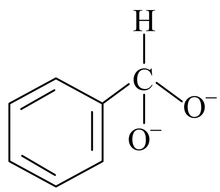
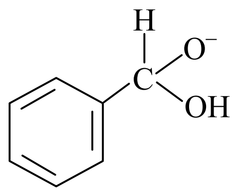
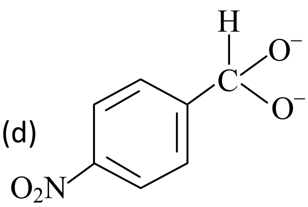
39.  $\xrightarrow{Ac_2O,\ H^+}$ (A) $\xrightarrow{AlCl_3}$ (B). Product (B) is :

(a)  (b)  (c)  (d) 

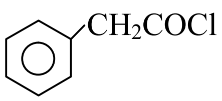
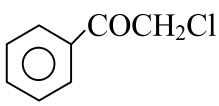
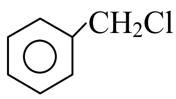
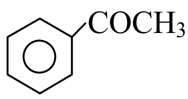
40. Which of the following will not undergo aldol condensation?

(a) Acetaldehyde
(b) Propionaldehyde
(c) Trideuterio acetaldehyde
(d) Benzaldehyde

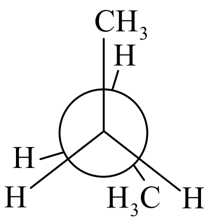
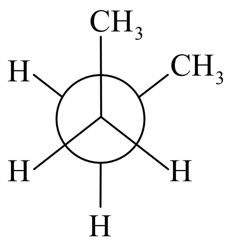
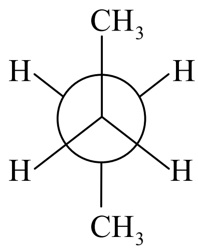
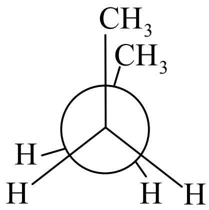
41. In the Cannizzaro reaction, the intermediate that will be the best hydride donor is

(a) 
(b) 
(c) 
(d) 

42. The following reaction is carried out. 
The major product formed is

(a) 
(b) 
(c) 
(d) 

43. In the following the most stable conformation of *n*-butane is

(a) 
(b) 
(c) 
(d) 

44. The number of S—S bonds in sulphur trioxide trimer S_3O_9 is:

(a) three
(b) two
(c) one
(d) zero

45. Nitrous oxide decomposes into N_2 and O_2 where reactants and products are in gas phase. If the reaction is first order then the rate constants for this reaction in terms of pressure i.e. P_i = initial pressure P_f = final pressure of reaction mixture may be denoted as:

(a) $k = \frac{1}{t}\ln\frac{P_i}{P_i - P_f}$
(b) $k = \frac{1}{t}\ln\frac{P_i}{P_f}$
(c) $k = \frac{1}{t}\ln\frac{P_i}{3P_i - 2P_f}$
(d) $k = \frac{1}{t}\ln\frac{P_i}{2P_f - 3P_i}$

46. Order of number of revolution/sec $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ for I, II, III and IV orbits is:

(a) $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4$ (b) $\gamma_4 > \gamma_3 > \gamma_2 > \gamma_1$

(c) $\gamma_1 > \gamma_2 > \gamma_4 > \gamma_3$ (d) $\gamma_2 > \gamma_3 > \gamma_4 > \gamma_1$

47. If the threshold frequency of a metal for photoelectric effect is ν_0, then which of the following will not happen?

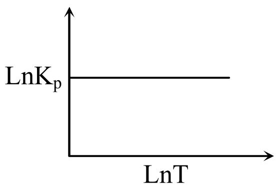
(a) If frequency of the incident radiation is ν_0, the kinetic energy of the electrons ejected is zero.

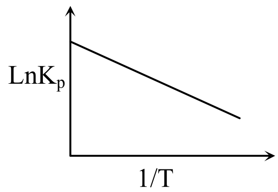
(b) If frequency of incident radiation is ν, the kinetic energy of the electrons ejected will be $h\nu - h\nu_0$.

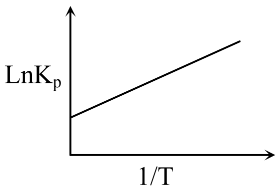
(c) If frequency is kept same at ν but intensity is increased, the number of electrons ejected will increase.

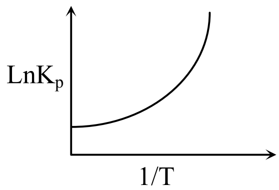
(d) If frequency of incident radiation is further increased, the number of photo–electrons ejected will increase.

48. An exothermic reaction is represented by the graph:

(a) 

(b) 

(c) 

(d) 

49. Which of the following composition shows maximum buffer capacity?

(a) 0.1 M CH_3COOH + 0.2 M CH_3COONa (b) 0.1 M CH_3COOH + 0.15 M CH_3COONa

(c) 0.05 M CH_3COOH + 0.15 M CH_3COONa (d) 0.1 M CH_3COOH + 0.12 M CH_3COONa

50. In a mixture of CH_3COOH and CH_3COONa, the ratio of salt to acid concentration is increased by ten folds. The pH of the solution will increase by:

(a) Zero (b) 1 (c) 2 (d) 3

51. Given that $E^0_{Cu^{2+}/Cu} = +0.34\text{ V}$ and $E^0_{Mg^{2+}/Mg} = -2.37\text{ V}$, which of the following is correct?

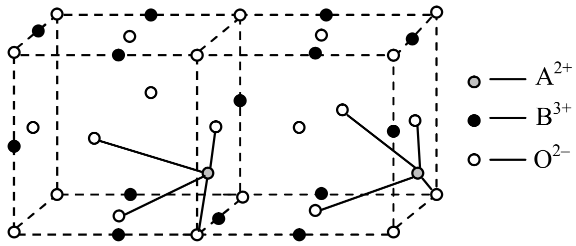
(a) Cu can oxidize H_2 into H^+. (b) Mg^{2+} can be reduced by H_2.

(c) Cu can reduce an Mg^{2+} ion. (d) Cu^{2+} can be reduced by H_2.

52. An aqueous solution containing an ionic salt having molality equal to 0.1892 freezes at – 0.704°C. The van't Hoff factor of the ionic salt is (given K_f for water = 1.86 K/m)

(a) 3 (b) 2 (c) 4 (d) 5

53. The formula of the compound is:



(a) ABO_2

(b) A_2BO_3

(c) AB_2O_4

(d) A_2BO_4

Answer the following question in reference to the above figure.

54. Given

$NH_{3(g)} + 3Cl_{2(g)} \rightleftharpoons NCl_{3(g)} + 3\ HCl; -\Delta H_1$

$N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}\ ; -\Delta H_2$

$H_{2(g)} + Cl_{2(g)} \rightleftharpoons 2HCl_{(g)}; -\Delta H_3$

The heat of formation of $NCl_{3(g)}$ in the terms of ΔH_1, ΔH_2 and ΔH_3 is

(a) $\Delta_f H = -\Delta H_1 + \frac{\Delta H_2}{2} - \frac{3}{2}\Delta H_3$ (b) $\Delta_f H = \Delta H_1 + \frac{\Delta H_2}{2} - \frac{3}{2}\Delta H_3$

(c) $\Delta_f H = \Delta H_1 - \frac{\Delta H_2}{2} - \frac{3}{2}\Delta H_3$ (d) None of these

55. A mixture of 2 mole of carbon monoxide gas and one mole of dioxygen gas is enclosed in a close vessel and is ignited to convert carbon monoxide into carbon dioxide. If the enthalpy change is ΔH and internal energy change is ΔU, then for the above process

(a) $\Delta H = \Delta U$ (b) $\Delta H + \Delta U = 1$

(c) $\Delta H - \Delta U > 0$ (d) $\Delta H < \Delta U$

Data required: Atomic Number Fe = 26, Zn = 30, Ni = 28, Cu = 29, Co = 27

56. Which of the following is not a π bonded complex?

(a) Zeise's salt
(b) Ferrocene
(c) Dibenzene chromium
(d) Tetraethyl lead

57. Which of the following complexes involves d^2sp^3 hybridization?

(a) $[FeF_6]^{3-}$
(b) $[Fe(CN)_6]^{3-}$
(c) $[Cr(NH_3)_6]^{3+}$
(d) $[Co(NH_3)_6]^{3+}$

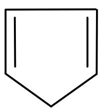
58. If a solution containing Al^{3+}, Ni^{2+} and Mg^{2+} is first treated with NH_4Cl and then with NH_4OH, which of the following will precipitate?

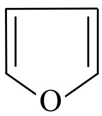
(a) $Al(OH)_3$
(b) $Ni(OH)_2$
(c) $Mg(OH)_2$
(d) $Al(OH)_3$, $Ni(OH)_3$ and $Mg(OH)_2$

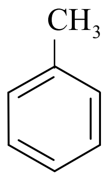
59. Which of the following leaves a black residue on the addition of NH_3?

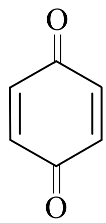
(a) AgCl
(b) $PbCl_2$
(c) Hg_2Cl_2
(d) $HgCl_2$

60. Which is maximum acidic?

(a) 

(b) 

(c) 

(d) 

MATHS

SECTION – III

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

61. Let $f(x)=\dfrac{ax^2+2x+1}{2x^2-2x+1}$. If $f:R\to[-1,\,2]$ is onto, then the value of a are

(a) $(-\infty,\,2)$ (b) $[2,\,\infty)$

(c) $(-\infty,\,-7]\cup[-2,\,\infty)$ (d) none of these

62. If the function $f(x)=\dfrac{(128a+ax)^{1/8}-2}{(32+bx)^{1/5}-2}$ is continuous at $x=0$, then the value of $\dfrac{a}{b}$ is

(a) $\dfrac{3}{5}f(0)$ (b) $2^{8/5}f(0)$ (c) $\dfrac{64}{5}f(0)$ (d) none of these

63. Let $f(x)$ be a polynomial function of second degree. If $f(1)=f(-1)$ and $a_1,\,a_2,\,a_3$ are in A.P., then $f'(a_1),\,f'(a_2),\,f'(a_3)$ are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

64. Let $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)};0<x<2,$ m and n are integers, $m\neq 0, n>0$, and let p be the left hand derivative of $|x-1|$ at $x=1$. If $\lim\limits_{x\to1^+}g(x)=p$, then:

(a) $n=1,m=1$ (b) $n=1,m=-1$ (c) $n=2,m=2$ (d) $n>2,m=n$

65. $\displaystyle\int\frac{dx}{x\sqrt{1-x^3}}=$

(a) $\dfrac{1}{3}\log\left|\dfrac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+c$ (b) $\dfrac{1}{3}\log\left|\dfrac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1}\right|+c$

(c) $\dfrac{1}{3}\log\left|\dfrac{1}{\sqrt{1-x^3}}\right|+c$ (d) $\dfrac{1}{3}\log\left|1-x^3\right|+c$

66. If $f(x)=\int_0^x \left(1+t^3\right)^{-1/2} dt$ and g is the inverse of f, then the value of $\frac{g''}{g^2}$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 1 (d) cannot be determined

67. Let $y(x)$ be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x,(x\geq 1)$. Then $y(e)$ is equal to:

(a) 0 (b) 2 (c) 2e (d) e

68. Line L has intercepts a and b on the coordinates axes When the axes are rotated through given angle, keeping the origin fixed, the same line L has intercepts p and q, then

(a) $a^2+b^2=p^2+q^2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$ (c) $a^2+p^2=b^2+q^2$ (d) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$

69. All chords of the curve $3x^2-y^2-2x+4y=0$ which subtend a right angle at the origin pass through the point

(a) $(1, 2)$ (b) $(-1, 2)$ (c) $(1, -2)$ (d) $(-1, -2)$

70. Let the straight line $x=b$ divides the area enclosed by $y=(1-x)^2$, $y=0$, and $x=0$ into two parts $R_1(0\leq x\leq b)$ and $R_2(b\leq x\leq 1)$ such that $R_1-R_2=\frac{1}{4}$. Then b equals

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

71. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x-y+1=0$ and $x-2y+3=0$, then the value of λ is

(a) 2 (b) 1 (c) –1 (d) –2

72. The shortest distance between the parabola $y^2=4x$ and the circle $x^2+y^2+6x-12y+20=0$ is

(a) $4\sqrt{2}-5$ (b) 0 (c) $3\sqrt{2}+5$ (d) 1

73. An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $P\left(\frac{1}{2}, 1\right)$. Its one directrix is the common tangent at the point P, to the circle $x^2+y^2=1$ and the hyperbola $x^2-y^2=1$. The equation of the ellipse in standard form is

(a) $9\left(x-\frac{1}{3}\right)^2+(y-1)^2=1$ (b) $9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$

(c) $\frac{\left(x-\frac{1}{3}\right)^2}{4}+\frac{(y-1)^2}{3}=1$ (d) none of these

74. For hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$ which of the following remains constant with change in 'α'

(a) abscissae of vertices (b) abscissae of foci (c) eccentricity (d) directirix

75. If M is a 3×3 matrix, where $M'M=I$ and det $M=1$, then det $(M-I)=$

(a) 0 (b) 1 (c) -1 (d) none of these

76. Let $\Delta(x)=\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}=Ax^4+Bx^3+Cx^2+Dx+E$. Then, the value of $5A+4B+3C+2D+E$ is equal to

(a) 9 (b) -9 (c) 11 (d) -11

77. The no. of real solutions of

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

(a) zero (b) one (c) two (d) infinite

78. If z_1 and z_2 both satisfy the relation $z+\overline{z}=2|z-1|$ and $\arg(z_1-z_2)=\frac{\pi}{4}$, then the imaginary part of (z_1+z_2) is

(a) 0 (b) 1 (c) 2 (d) none of these

79. The number of 7 digit numbers the sum of whose digits is even, is

(a) 35×10^5 (b) 45×10^5 (c) 50×10^5 (d) none of these

80. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is :

(a) 4 (b) $\frac{6}{25}$ (c) $\frac{12}{5}$ (d) 6

81. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ such that $\min.f(x)>\max.g(x)$, then the relation between b and c is

(a) $|c|<|b|\sqrt{2}$ (b) $0<c<b\sqrt{2}$ (c) $|c|<|b|\sqrt{2}$ (d) $|c|>|b|\sqrt{2}$

82. If $ax^2+bx+6=0$ does not have two distinct real roots $a\in R$, $b\in R$, then the least value of $3a+b$ is

(a) 4 (b) –1 (c) 1 (d) –2

83. If $A={}^{2n}C_0\,{}^{2n}C_1+{}^{2n}C_1\,{}^{2n-1}C_1+{}^{2n}C_2\,{}^{2n-2}C_1+\ldots$, then A is

(a) 0 (b) 2^n (c) $n2^{2n}$ (d) 1

84. The mean of 10 numbers is 12.5, the mean of the first six is 15 and the last five is 10. The sixth number is

(a) 12 (b) 15 (c) 18 (d) None of these

85. The number of values of x in the Interval $(0, 5\pi)$ satisfying equation $3\sin^2 x - 7\sin x + 2 = 0$ is

(a) 0 (b) 5 (c) 6 (d) 10

86. A solution of the equation $(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$, where θ lies in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is given by

(a) $\theta = 0$ (b) $\theta = \frac{\pi}{3}$ or $-\frac{\pi}{3}$ (c) $\theta = \frac{\pi}{6}$ (d) $\theta = -\frac{\pi}{6}$

87. If $a = \hat{i} + \hat{j} + \hat{k}$, $b = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $c = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly depended vectors and $|c| = \sqrt{3}$, then

(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 1, \beta = \pm 1$ (c) $\alpha = -1, \beta \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

88. If $f(x) = \begin{cases} e^{\cos x}\sin x , & \text{for} |x| \le 2 \\ 2 , & \text{otherwise} \end{cases}$ then $\int_{-2}^{3} f(x)\,dx$ is

(a) 0 (b) 1 (c) 2 (d) 3

89. Equation of the projection of the line $8x - y - 7z = 8$, $x + y + z = 1$ on the plane $5x - 4y - z = 5$ is

(a) $\frac{x-1}{1} = \frac{y}{2} = \frac{z}{-3}$ (b) $\frac{x}{1} = \frac{y-1}{2} = \frac{z}{-3}$ (c) $\frac{x}{1} = \frac{y}{2} = \frac{z-1}{-3}$ (d) $\frac{x}{1} = \frac{y+1}{-2} = \frac{z+1}{3}$

90. The cartesian equation of the plane

$\mathbf{r} = (1+\lambda-\mu)\mathbf{i} + (2+\lambda)\mathbf{j} + (3-2\lambda+2\mu)\mathbf{k}$ is

(a) $2x + y = 5$ (b) $2x - y = 5$ (c) $2x + z = 5$ (d) $2x - z = 5$

SOLUTION OF AITS JEE (MAIN) FULL TEST – 2

PHYSICS

1. **(a)**

$-\dfrac{\alpha z}{k\theta}$ is dimension less quantity

dimension of $\alpha = \dfrac{k\theta}{z} = \dfrac{ML^2T^{-2}K^{-1}}{L} \times K$

$\alpha = MLT^{-2}$

Dimension of $\dfrac{\alpha}{\beta}$ is equal to dimension of pressure P

$P = \left(\dfrac{\alpha}{\beta}\right)$

$ML^{-1}T^{-2} = \dfrac{MLT^{-2}}{\beta}$

$\beta = \dfrac{MLT^{-2}}{ML^{-1}T^{-2}}$

$\beta = M^0L^2T^0$

2. **(c)**

The displacement of the body during the time t as it comes back to the point of projection

$\Rightarrow\ S = 0\ \Rightarrow\ v_0 t - \dfrac{1}{2}gt^2 = 0 \Rightarrow t = \dfrac{2v_0}{g}$

During the same time t, the body moves in absence of gravity through a distance D′ = v.t, because in absence of gravity g = 0

$\Rightarrow\ D' = v_0 \times \left(\dfrac{2v_0}{g}\right) = \dfrac{2v_0^2}{g}$ …(1)

In presence of gravity the total distance covered is

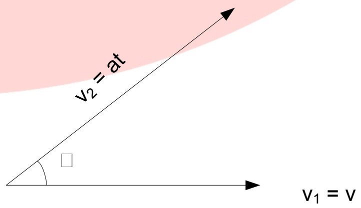
$= D = 2H = 2\dfrac{v_0^2}{2g} = \dfrac{v_0^2}{g}$ …(2)

(A) ÷ (B) ⇒ D′ = 2D

3. **(b)**

Velocity of 1st particle at time t is $v_1 = v$

Velocity of 2nd particle at time t is $v_2 = at$



Relative velocity $v_r = \sqrt{v_2^2 - 2v_1v_2\cos\alpha}$

$\Rightarrow\ v_r^2 = v^2 + a^2t^2 - 2v(at)\cos\alpha$

For v_r to be least or v_r^2 to be least we must have

$$\frac{d}{dt}(v_r^2) = 0$$

$$\Rightarrow\ 0 + 2a^2t - 2av\cos\alpha = 0 \ \Rightarrow\ t = \frac{v\cos\alpha}{a}$$

4. **(b)**

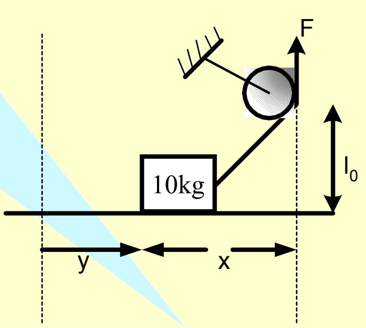
Accel. of $\frac{mg\sin^2 30}{m} = g\sin^2 30$

$s = 5 = \frac{1}{2}\times 4\times t^2 \ \Rightarrow\ t = \sqrt{\frac{10\times 4}{10}} = 2\,\text{sec}$

5. **(b)**

In this case force F is variable

Common mistake: $\vec{F} = \left(F\cos\theta\,\hat{i} + F\sin\theta\,\hat{j}\right)$



$dw = \vec{F}.dx\,i$

$= [F\cos\theta\, i + F\sin\theta\, j].\, dx\, i$

$= F\cos\theta\ dx$

$\cos\theta = \frac{x}{\sqrt{l_0^{\,2} + x^2}}$

$$\int dw = \int \frac{Fx}{\sqrt{l_0^{\,2}+x^2}}dx = F\left[\sqrt{l_0^{\,2}+x^2}\right]_{25}^{20} < 0$$

……………………………………

We know that in this case angle between $\vec{F}$ and displacement is acute so work should be positive, contradiction has come.

$y = 25 - x \Rightarrow dy = -dx$ and

$dw = (F\cos\theta\, i + F\sin\theta\, j).\, dy\, i$

$dw = -F\cos\theta\, dx$

$$w = -\int_{25}^{20} \frac{Fx}{\sqrt{l_0^{\,2}+x^2}}dx = F\left[\sqrt{l_0^{\,2}+x^2}\right]_{20}^{25} = F\left[\sqrt{626} - \sqrt{401}\right]J$$

6. **(b)**

Mass of water pumped per second

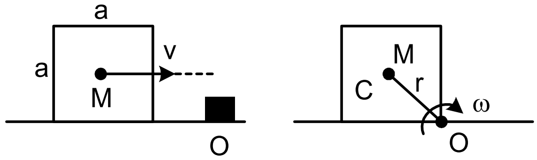
$m = \frac{2400}{60} = 40\,kg$

$v = 3\,m/s$

Kinetic energy of water coming out per second $= \frac{1}{2}mv^2 = \frac{1}{2}\times 40\times 3\times 3 = 180\,J$

$\therefore$ Power of pump = 180 J /s = 180 W

7. **(a)**



$r = \sqrt{2}\dfrac{a}{2}$ or $r^2 = \dfrac{a^2}{2}$

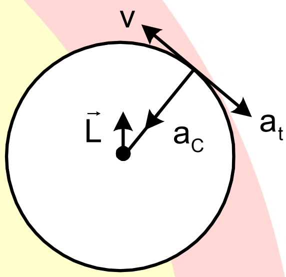
Net torque about O is zero.

Therefore, angular momentum (L) about O will be conserved, or $L_i = L_f$

$$MV\left(\frac{a}{2}\right) = I_0\omega = (I_{cm} + Mr^2)\omega = \left\{\left\{\frac{Ma^2}{6} + M\left(\frac{a^2}{2}\right)\right\}\right\}\omega$$

$$\omega = \frac{3V}{4a}$$

8. **(b)**



$|\vec{L}| = mvr$

Since v is changing (decreasing), L is not conserved in magnitude. Since it is given that a particle is confined to rotate in a circular path, it cannot have spiral path. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.

The direction of $\vec{L}$ remains same even when the speed decreases.

9. **(c)**

Let M be the mass of the planet and m the mass of satellite. Then

$$mr\omega^2 = \frac{GMm}{r^2} \Rightarrow GM = r^3\omega^2$$

Now $g = \dfrac{GM}{R^2} = \dfrac{r^3\omega^2}{R^2}$

10. **(d)**

Pressure on bottom = $P_{at} + h\rho g$.

Pressure on top = P_{at} d

Average pressure on the wall = $\dfrac{P_{at} + (P_{at} + h\rho g)}{2}$

Using Pascal's law, average pressure on the wall $P_{at} + \dfrac{1}{2}\rho g H$

Area of the wall in contact with water = $\dfrac{\omega h}{\cos\theta}$

Force exerted on wall is $F = P_{av}\,A = \left(P_{at} + \dfrac{1}{2}\rho g H\right)\dfrac{\omega H}{\cos\theta}$

11. **(b)**

After P length of pendulum becomes $\dfrac{l}{4}$

Now as $T \propto \sqrt{l}$, so after P time period will become T' = T/2, Therefore, the desired time will be:

$$= \frac{T}{2} + \frac{T'}{2} = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

12. **(a)**

$\therefore$ Distance covered = v.5 + $\frac{1}{2}$ a.5^2 = 5[v+ (5a/2)]

$\because$ Number of positive crests striking per second is same as the frequency.

$\therefore$ (a)

13. **(c)**

Change in internal energy depends only on change in temperature since internal energy is a function of state only i.e. dU = nCv, dT.

In adiabatic process, dQ = 0,

Hence, dU + dW = 0 $\Rightarrow$ dU = – dW

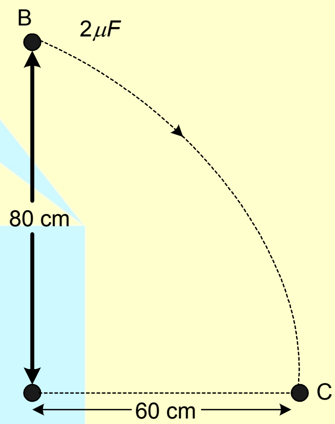
i.e. magnitude of change in internal energy is equal to magnitude of work done.

14. **(d)**

According to Kirchoff's law good emitter is a good absorber

$\therefore$ $E_y > E_x$ and $A_y > A_x$

15. **(d)**



$$w = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

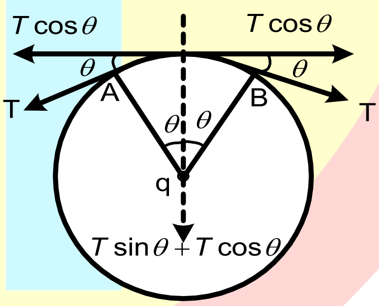
$$= 10 \times 2 \times 10^{-12} \times 9 \times 10^9 \left[\frac{1}{0.6} - \frac{1}{0.8}\right]$$

= 0.075 J

16. **(d)**

Consider a small element AB, θ is very small. Then

AB = R (2θ)



Charge on AB is $dQ = \frac{Q}{2\pi R}(2R\theta) = \frac{Q\theta}{\pi}$

$$2T \sin\theta = \frac{dQ \cdot q}{4\pi \in_0 R^2} = \frac{Qq\theta}{4\pi^2 \in_0 R^2}$$

$$2T\theta = \frac{Qq\theta}{4\pi^2 \in_0 R^2} \text{ or } T = \frac{Qq}{8\pi^2 \in_0 R^2}$$

17. **(a)**

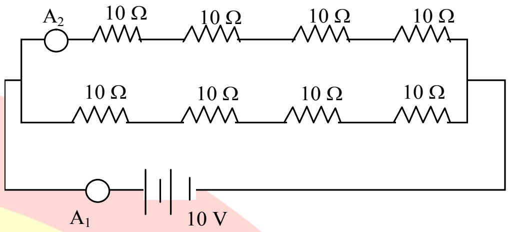
$$f = \frac{v}{2\pi r} = \frac{2.2 \times 10^6 \text{ m/s}}{2\pi(5.3 \times 10^{-11} \text{m})} = 6.6 \times 10^{15} \text{ rev/s}$$

Each time the electron goes around the orbit, it carries a charge e around the loop. The charge passing a point on the loop each second is

Current = $I = ef = (1.6 \times 10^{-19}\text{C})(6.6 \times 10^{15}\text{s}^{-1}) = 1.06$ mA. Note that the current flows in the opposite direction to the electron, which is negatively charged.

18. **(c)**

Potential difference across R_2 resistances is zero, therefore current in three branches is zero, therefore current in two branch containing R_1 will be same, simplified circuit will be



Effective resistance of the circuit $R_{eff} = \frac{40 \times 40}{40 + 40} = 20\ \Omega$

current through the circuit $I = \frac{10}{20} = (1/2)$ amp

Hence reading of $A_1 = 1/2$ amp.

Hence reading of $A_2 = 1/4$ amp.

$\therefore$ **(c)**

19. **(b)**

$$\sqrt{\frac{M_A}{M_B}} = \frac{T_B}{T_A} \quad \text{or} \quad T_B = T \times \sqrt{\frac{M_A}{4M_A}} = \frac{T}{2}$$

20. **(c)**

The magnetic lines of force in a ferrmagnetic material are crowded together.

21. **(b)**

$$\varpi' = \sqrt{\frac{1}{L/2.C/2}} = \sqrt{\frac{4}{LC}} = 2\left(\sqrt{\frac{1}{LC}}\right) = 2\varpi$$

22. **(d)**

According to the problem, the e.m.f. induced in an elementary ring of radius r and width dr is $\pi r^2 \beta$.

The conductance of this ring is $d\left(\frac{1}{R}\right) = \left(\frac{h\,dr}{p\,2\pi r}\right)$

thus, the current induced is $dI = hrdr\beta / 2p$. Upon integration we get the total current

$$I = \int_a^b \frac{hrdr\beta}{2p} = \frac{h\beta(b^2 - a^2)}{4p}$$

23. **(a)**

$$\frac{P_a}{P_l} = \frac{\left(\frac{\mu_g}{\mu_a} - 1\right)}{\left(\frac{\mu_g}{\mu_l} - 1\right)} = \frac{5}{-100/100} = -5 \Rightarrow \mu_l = \frac{5}{3}$$

24. **(b)**

Since there is no shift in central maxima. Therefore path difference introduced by the two sheets are equal i.e. $(\mu_1 - 1)\,t_1 = (\mu_2 - 1)\,t_2$

where μ_1 and μ_2 are refraction index

i.e. $\frac{t_1}{t_2} = \frac{(\mu_2 - 1)}{(\mu_1 - 1)} = \frac{(1.5 - 1)}{(1.25 - 1)} = \frac{0.5}{0.25} = 2$

25. **(d)**

$N_1 = N_0\ e^{-10\lambda t}$

$N_2 = N_0\ e^{-\lambda t}$

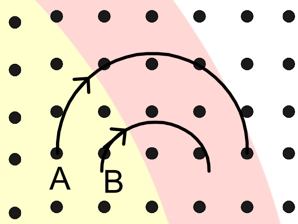
$\Rightarrow 9\lambda t = 1$, or $t = \dfrac{1}{9\lambda}$

26. **(a)**

$A_I = \dfrac{A_V}{A_R} = \dfrac{2800}{3000} = 0.93$

27. **(b)**

When a charged particle is moving at right angle to the magnetic field than a force acts on it which behaves as a centripetal force and moves the particle in circular motion.



$\therefore\ \dfrac{m_A v_A^2}{2r} = q.v_A B$

$\therefore\ \dfrac{m_A v_A}{2r} = qB$

Similarly for second particle moving with half radius as compared to first we have

$\dfrac{m_B v_B}{r} = qB \Rightarrow \dfrac{m_A v_A}{2r} = \dfrac{m_B v_B}{r} \Rightarrow m_A\ v_A = 2m_B v_B \Rightarrow m_A v_A > m_B v_B$

28. **(d)**

$I \propto \dfrac{1}{d^2}$

On doubling the distance the intensity becomes one fourth i.e. only one fourth of photons now strike the target in comparison to the previous number. Since photoelectric effect is a one photon-one electron phenomena, so only one-fourth photoelectrons are emitted out of the target hence reducing the current to one fourth the previous value.

29. **(c)**

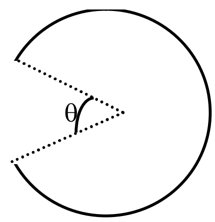
Work Function $\phi_0 = h\nu_0 = h\nu_0 = \dfrac{hc}{\lambda_{max.}}$

$\therefore\ \lambda_{max.} = \dfrac{hc}{\phi_0} = \dfrac{1240 eV.nm}{4eV} = 310nm$

30. **(b)**

Electric field due to an arc at its centre is

$\dfrac{k\lambda}{R} 2\sin\left(\dfrac{\theta}{2}\right)$, Where $k = \dfrac{1}{4\pi\varepsilon_0}$, θ = angle subtended by the wire at the centre,



λ = Linear density of charge.

Let E be the electric field due to remaining portion.

Since intensity at the centre due to the circular wire is zero.

Applying principle of superposition.

$$\frac{k\lambda}{R}2\sin\left(\frac{\theta}{2}\right)\hat{n}+\vec{E}=0$$

$$\left|\vec{E}\right|=\frac{1}{4\pi\varepsilon_0 R}\cdot\frac{Q}{2\pi R}.2\sin\left(\frac{\theta}{2}\right)$$

$$=\frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{\theta}{2}\right)$$

CHEMISTRY

31. **(b)**

32. **(b)**

33. **(c)**

There are 2 chiral centres and 1 double bond capable of showing geometrical isomerism.

$2^2 \times 2^1 = 8$

34. **(d)**

The two methyl groups are at farthest position. In the chair form, one methyl occupies axial position while other methyl group will occupy equatorial position.

35. **(d)**

36. **(c)**

37. **(a)**

38. **(a)**

39. **(c)**

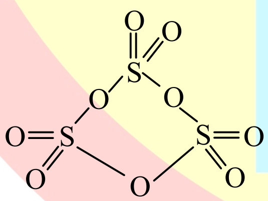
40. **(d)**

41. **(b)**

42. **(b)**

43. **(c)**

44. **(d)**

S_3O_9 is 

45. **(c)**

$2N_2O(g) \longrightarrow 2N_2(g) + O_2(g)$

P_i 0 0

$P_i - 2P$ 2P P

For a first order reaction,

$$k=\frac{1}{t}\ln\frac{P_i}{P_i-2P}$$

Where P_i = initial pressure of gas

$P_i - 2P$ = pressure of gas at time t.

Total pressure at t = $P_i + P = P_f$

$P = P_f - P_i$

$\therefore$ $P_i - 2P = P_i - 2P_f + 2P_i = 3P_i - 2P_f$

$$k = \frac{1}{t}\ln\frac{P_i}{3P_i - 2P_f}$$

46. **(a)**

Lower is the shell number more is the velocity of an electron and lesser is the circumference of the electron. Hence, more are the number of revolutions.

47. **(d)**

48. **(c)**

For exothermic reactions, increasing 'T' will decrease the value of $'K_P'$

49. **(d)**

The buffer whose pH is more closer to pK_a will have more buffer capacity.

50. **(b)**

$pH = pK_a + \log\frac{[Salt]}{[Acid]}$ If $\frac{[Salt]}{[Acid]}$ increases by 10 times pH = pK_a + 1

51. **(d)**

$Mg \rightarrow Mg^{2+} + 2e^-$

$\frac{Cu^{2+} + 2e^- \rightarrow Cu}{Mg + Cu^{2+} \rightarrow Mg^{2+} + Cu}$. Hence, Cu cannot reduce Mg^{2+} ion.

Decreasing order of reduction potential is as follows: $Cu^{+2} > H^+ > Mg^{+2}$

52. **(a)**

because it is equal to the mole fraction of solute.

53. **(c)**

Number of oxide ions $= \frac{1}{8} \times 8$ corners $+ \frac{1}{2} \times 6$ face-centres = 4

Number of A^{2+} ions present in tetrahedral void = 1

Number of B^{3+} ions = 2

$\therefore$ Formula of compound = AB_2O_4

54. **(d)**

$$\frac{1}{2}N_2(g) + \frac{3}{2}Cl_2(g) \rightarrow NCl_3 \; \Delta H_f = ?$$

$\Delta H_f = \Sigma\Delta H_f$, pds. $- \Sigma\Delta H_f$, reactants = $-\Delta H_1 + \frac{1}{2}\Delta H_2 + \frac{3}{2}\Delta H_3$

55. **(d)**

$1\ CO + \frac{1}{2}O_2 \rightarrow 1\ CO_2$; ΔH

2 mol 1 mole

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = \Delta U - \frac{RT}{2}$

$\Rightarrow \Delta U > \Delta H$

56. **(d)**

TEL is sigma bonded complex.

57. **(d)**

58. **(a)**

59. **(c)**

$Hg_2Cl_2 + 2NH_4OH \rightarrow \underbrace{Hg + Hg(NH_2)Cl}_{\text{black}} + NH_4Cl + 2H_2O$

60. **(a)**

MATHS

61. **(c)**

We have,

$$f(x) = \frac{ax^2 + 2x + 1}{2x^2 - 2x + 1}$$

which is defined $\forall\ x \in R$, since

$$2x^2 - 2x + 1 = 2\left(x - \frac{1}{2}\right)^2 + \frac{1}{2} \neq 0 \text{ for any real } x.$$

Now, if $f : R \rightarrow [-1, 2]$ is onto, then

$$-1 \leq \frac{ax^2 + 2x + 1}{2x^2 - 2x + 1} \leq 2,\ \forall\ x \in R$$

Solving the left-hand inequality, we have

$$(a+2)x^2 + 2 \geq 0$$

which is true $\forall\ x \in R$ if $a \geq -2$ … (1)

Solving the right-hand inequality, we have

$$(a-2)x^2 + 6x - 1 \leq 0$$

which is true for all $x \in R$ if coefficient of $x^2 < 0$ and $D \leq 0$

i.e., $a < 2$ and $36 + 4(a-2) \le 0$

i.e., $a < 2$ and $a \le -7$

i.e., $a \le -7$ … (2)

Hence, from (1) and (2) the permissible values of a are given by

$$(-\infty, -7] \cup [-2, \infty)$$

62. **(c)**

If f is continuous at $x = 0$, then

$$f(0) = \lim_{x \to 0} \frac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$$

As $x \to 0$, the denominator $\to 0$. Thus, for limit to exist the numerator must also $\to 0$. Thus, we have

$$(128a)^{1/8} = 2$$

gives $a = 2$

Now, we have,

$$f(0) = \lim_{x \to 0} \frac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2} \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 0} \frac{\frac{2}{8}(256 + 2x)^{-7/8}}{\frac{b}{5}(32 + bx)^{-4/5}} = \frac{5}{4b} \cdot \frac{2^{-7}}{2^{-4}} = \frac{5}{32b}$$

gives $b = \dfrac{5}{32 f(0)}$

Hence, we have,

$$\frac{a}{b} = \frac{64}{5} f(0)$$

63. **(a)**

Let $f(x) = ax^2 + bx + c$

Then, $f'(x) = 2ax + b$

Also, $f(1) = f(-1)$

$\Rightarrow a + b + c = a - b + c \Rightarrow 2b = 0 \Rightarrow b = 0$

$\therefore f'(x) = 2ax$

$\therefore f'(a_1) = 2aa_1,\ f'(a_2) = 2aa_2$ and $f'(a_3) = 2aa_3$

As a_1, a_2, a_3 are in A.P., we have

$f'(a_1), f'(a_2), f'(a_3)$ are in A.P.

64. **(c)**

65. **(a)**

$$I=\int x^{-1}\left(1-x^3\right)^{-1/2}dx$$

Let $1-x^3=t^2 \Rightarrow -3x^2dx=2t\ dt \ \Rightarrow \ x^{-1}dx=-\frac{2}{3}\frac{tdt}{x^3}=-\frac{2}{3}\frac{tdt}{1-t^2}$

$$\therefore \quad I=\int\left(t^{-1}\right)\left(\frac{-2}{3}\right)\frac{tdt}{1-t^2}=-\frac{2}{3}\int\frac{dt}{1-t^2}=\frac{2}{3}\int\frac{dt}{t^2-1}=\frac{2}{3}\cdot\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c=\frac{1}{3}\ln\left(\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|\right)+c$$

66. **(b)**

We have,

$$f(x)=\int_0^x\left(1+t^3\right)^{-1/2}dt \Rightarrow f\left(g(x)\right)=\int_0^{g(x)}\left(1+t^3\right)^{-1/2}dt$$

$$\Rightarrow \quad x=\int_0^{g(x)}\left(1+t^3\right)^{-1/2}dt \qquad [g \text{ is inverse of } f \Rightarrow f\{g(x)\}=x]$$

Differentiating w.r.t. x, we have

$$1=\left(1+g^3\right)^{-1/2}\cdot g'$$

i.e., $\quad (g')^2=1+g^3$

Differentiating again w.r.t. x, we have

$$2g'g''=3g^2g' \Rightarrow \frac{g''}{g^2}=\frac{3}{2}.$$

67. **(b)**

68. **(b)**

69. **(c)**

Let $lx+my=1$ $\qquad$ ... (1)

be any chord of the curve $3x^2-y^2-2x+4y=0,$ $\qquad$ ... (2)

which subtends a right angle at the origin.

Making (2) homogeneous with the help of (1), we obtain

$$3x^2-y^2-2x(lx+my)+4y(lx+my)=0 \text{ or, } (3-2l)x^2+(4m-1)y^2+2(2l-m)xy=0.$$

These angles are right angles, if

$$(3-2l)+(4m-1)=0 \text{ or } l-2m=1,$$

which shows that (1) passes through the fixed point $(1,-2)$.

70. **(b)**

71. **(a)**

Let the lines cuts the x-axis at A and B, then

$$OA = -\frac{1}{\lambda} \text{ and } OB = -3.$$

Also, if the lines cut the y-axis at C and D, then

$$OC = 1 \text{ and } OB = \frac{3}{2}.$$

Now, if the circle passes through A, B, C and D then

$$OA \times OB = OC \times OD \Rightarrow \left(-\frac{1}{\lambda}\right)(-3) = 1 \times \frac{3}{2} \Rightarrow \lambda = 2.$$

72. **(a)**

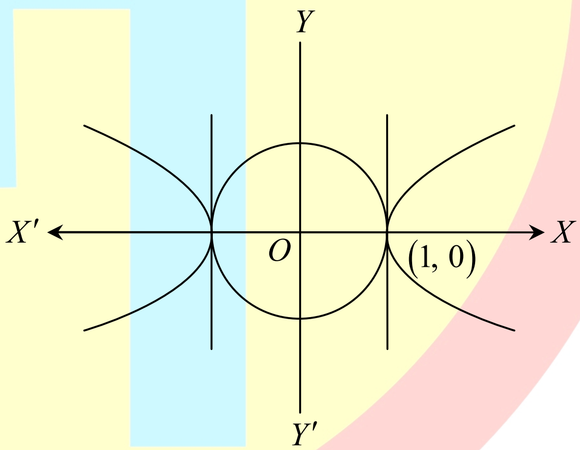
Normal at a point $(m^2, -2m)$ on the parabola $y^2 = 4x$ is given by $y = mx - 2m - m^3$. If this is normal to the circle also, then it will pass through centre of the circle so

$$6 = -3m - 2m - m^3 \Rightarrow m = 1$$

Since shortest distance between parabola and circle will occur along common normal, shortest distance is $4\sqrt{2} - 5$.

73. **(b)**

Clearly, the common tangent to the circle $x^2 + y^2 = 1$ and hyperbola $x^2 - y^2 = 1$ is $x = 1$ [which is nearer to $P(1/2, 1)$].



Given, one focus at $P\left(\frac{1}{2}, 1\right)$.

$\therefore$ equation of the directrix is $x = 1$.

$\therefore$ ellipse is $\sqrt{\left(x - \frac{1}{2}\right)^2 + (y-1)^2} = \frac{1}{2}(x-1)$

On simplification, it becomes

$$9\left(x - \frac{1}{3}\right)^2 + 12(y-1)^2 = 1.$$

74. **(b)**

75. **(a)**

$\det M = 1 \Rightarrow \det M' = 1$

Now, $\det(M - I) = \det(M - I)\det M'$

$$= \det(MM' - IM') \qquad (\because \det A \det B = \det(AB))$$

$$= \det(I - M') = -\det(M' - I) = -\det(M - I)$$

Thus, $\det(M - I) = -\det(M - I) \Rightarrow \det(M - I) = 0.$

76. **(d)**

$$\Delta'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$$

$$\therefore\ 5A + 4B + 3C + 2D + E = \Delta(1) + \Delta'(1)$$

But, $\Delta(1) = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{vmatrix} = 0$ $\qquad [\because R_2, R_3 \text{ are identical}]$

$$\Delta'(x) = \begin{vmatrix} 1 & 0 & 1 \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ 2x & 1 & 0 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\therefore\ \Delta'(1) = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 0 + \begin{vmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & -1 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 0 & 0 & -6 \end{vmatrix} = -17 - 6(1-2) = -17 + 6 = -11$$

$\therefore$ required value $= -11$.

77. **(c)**

78. **(c)**

Let $z = x + iy$

We have, $z + \bar{z} = 2|z-1|$

$$\Rightarrow \frac{z+\bar{z}}{2} = |z-1| \Rightarrow x = |x + iy - 1| \Rightarrow x = |(x-1) + iy|$$

$$\Rightarrow x^2 = (x-1)^2 + y^2 \Rightarrow 2x = 1 + y^2.$$

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

then, $\quad 2x_1 = 1 + y_1^2 \qquad \ldots (1)$

and, $\quad 2x_2 = 1 + y_2^2 \qquad \ldots (2)$

Subtracting (2) from (1), we get

$$2(x_1 - x_2) = y_1^2 - y_2^2$$

$$\Rightarrow 2(x_1 - x_2) = (y_1 + y_2)(y_1 - y_2) \qquad \ldots (3)$$

But given $\arg(z_1 - z_2) = \dfrac{\pi}{4}$

i.e., $\quad \tan^{-1}\left(\dfrac{y_1 - y_2}{x_1 - x_2}\right) = \dfrac{\pi}{4} \Rightarrow \dfrac{y_1 - y_2}{x_1 - x_2} = 1$

$\therefore \quad y_1 - y_2 = x_1 - x_2, \qquad \ldots (4)$

$\therefore$ From (3) and (4) we get

$$y_1 + y_2 = 2$$

$\therefore$ $\text{Im}(z_1 + z_2) = 2.$

79. **(b)**

A number of the seven digits will be of the form.

$$ax_1x_2x_3x_4x_5 0,\ ax_1x_2x_3x_4x_5 1,\ ax_1 x_2x_3x_4x_5 2,\ ax_1 x_2x_3x_4x_5 3,\ \dots\ ax_1 x_2x_3x_4x_5 9$$

where $a \in \{1, 2, 3, \dots 9\}$

and $x_1, x_2, x_3, x_4, x_5 \in \{0, 1, 2, 3, \dots 9\}$.

Since sum of the digits should be even, therefore, if $a + x_1 + x_2 + x_3 + x_4 + x_5$ is an even number, then the digit at units place must be 0, 2, 4, 6, 8 and if $a + x_1 + x_2 + x_3 + x_4 + x_5$ is an odd number, then the digit at units place will be 1, 3, 5, 7, 9.

$\therefore$ Required number $= 9 \times 10 \times 10 \times 10 \times 10 \times 10 \times 5 = 9 \times 10^5 \times 5 = 45 \times 10^5$.

80. **(c)**

81. **(d)**

$$\min.\ f(x) = -\frac{D}{4a} = -\frac{4b^2 - 8c^2}{4} = -(b^2 - 2c^2) \quad \text{(upward parabola)}$$

$$\max.\ g(x) = -\frac{D}{4a} = \frac{4c^2 + 4b^2}{4} = b^2 + c^2 \quad \text{(downward parabola)}$$

Now, $2c^2 - b^2 > b^2 + c^2 \Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$

82. **(d)**

Since $ax^2 + bx + 6 = 0$ does not have two distinct real roots

$\therefore$ $b^2 - 24a \le 0$

Let $3a + b = y$ $\quad \therefore$ $3a = y - b$

$\therefore$ $b^2 - 8(y - b) \le 0$ i.e., $b^2 + 8b - 8y \le 0$

Since b is real $\therefore 64 + 32y \ge 0 \Rightarrow y \ge -2$

$\therefore$ Min. value of y, i.e., $3a + b = -2$.

83. **(c)**

$A =$ coefficient of x in $\left[{}^{2n}C_0 (1+x)^{2n} + {}^{2n}C_1 (1+x)^{2n-1} + \dots \right]$

$=$ coefficient of x in $(1 + (1+x))^{2n}$ $=$ coefficient of x in $(2+x)^{2n}$

$=$ coefficient of x in $2^{2n}\left(1 + \frac{x}{2}\right)^{2n} = n \cdot 2^{2n}$

84. **(b)**

85. **(c)**

86. **(b)**

We have, $(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0 \Rightarrow \left(1-\tan^4\theta\right) + 2\tan^2\theta = 0$

Put $\tan^2\theta = x$, $\therefore \left(1-x^2\right) + 2^x = 0 \Rightarrow 2x = x^2 - 1 = y(\text{say})$

$\therefore \quad y = 2^x$ and $y = x^2 - 1$

By inspection, $x = 3$, $\therefore \tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3} \Rightarrow \theta = \pm\dfrac{\pi}{3}$

87. **(d)**

88. **(c)**

89. **(a)**

Any plane through the given line is

$$x + y + z - 1 + \lambda(8x - y - 7z - 8) = 0 \qquad \ldots (1)$$

$\Rightarrow (1+8\lambda)x + (1-\lambda)y + (1-7\lambda)z - 1 - 8\lambda = 0$

It is perpendicular to the plane $5x - 4y - z = 5$

If $(1+8\lambda)5 + (1-\lambda)(-4) - (1-7\lambda) = 0 \Rightarrow \lambda = 0$

So that (1) becomes $x + y + z - 1 = 0$

Now, the line of intersection of the planes $x + y + z - 1 = 0$ and $5x - 4y - z = 5$ is the required line of projection, which clearly passes through $(1, 0, 0)$ and if l, m, n are direction ratios of the line then $l + m + n = 0$ and $5l - 4m - n = 0$

$$\Rightarrow \frac{l}{-1+4} = \frac{m}{5+1} = \frac{n}{-4-5} \Rightarrow \frac{l}{3} = \frac{m}{6} = \frac{n}{-9} \Rightarrow \frac{l}{1} = \frac{m}{2} = \frac{n}{-3}$$

and hence the required equation of the projection is

$$\frac{x-1}{1} = \frac{y}{2} = \frac{z}{-3}$$

90. **(c)**

We have, $\mathbf{r} = (1+\lambda-\mu)\mathbf{i} + (2-\lambda)\mathbf{j} + (3-2\lambda+2\mu)\mathbf{k}$

$\Rightarrow \mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{k})$.

Which is a plane passing through

$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and parallel to the vectors

$\mathbf{b} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + 2\mathbf{k}$.

Therefore, it is $\perp$ to the vector

$\mathbf{n} = \mathbf{b} \times \mathbf{c} = -2\mathbf{i} - \mathbf{k}$.

Hence, its vector equation is $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$

$\Rightarrow \mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n} \Rightarrow \mathbf{r}.(-2\mathbf{i} - \mathbf{k}) = -2 - 3 \Rightarrow \mathbf{r} \cdot (2\mathbf{i} - \mathbf{k}) = 5$

So, the cartesian equation is $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (2\mathbf{i} + \mathbf{k}) = 5$ or, $2x + z = 5$.